~~arbitrarily large number of bundles. This is summarized by the following "strong" axiom:~~

~~**DEFINITION**~~

~~**Strong Axiom of Revealed Preference** The *strong axiom of revealed preference* states that if commodity bundle 0 is revealed preferred to bundle 1, and if bundle 1 is revealed preferred to bundle 2, and if bundle 2 is revealed preferred to bundle 3, . . . , and if bundle $K - 1$ is revealed preferred to bundle K, *then* bundle K cannot be revealed preferred to bundle 0 (where K is any arbitrary number of commodity bundles).~~

~~Most other properties that have been developed using the concept of utility can be proved using this revealed preference axiom instead. For example, it is an easy matter to show that demand functions are homogeneous of degree zero in all prices and income. It therefore is apparent that the revealed preference axiom and the existence of "well-behaved" utility functions are somehow equivalent conditions. That this is in fact the case was first shown by H. S. Houthakker in 1950. Houthakker showed that a set of indifference curves can always be derived for an individual who obeys the strong axiom of revealed preference.[10] Hence, this axiom provides a quite general and believable foundation for utility theory based on relatively simple comparisons among alternative budget constraints. This approach is widely used in the construction of price indices and for a variety of other applied purposes.~~

CONSUMER SURPLUS

An important problem in applied economics is to develop a monetary measure of the gains or losses that individuals experience as a result of price changes. For example, as we will show in Part VI, if sellers of a commodity are relatively few in number, they may be able to raise the market price of the commodity in order to obtain greater profits. To put a monetary cost on this distortion, we need some way of evaluating the welfare loss that consumers experience from the price rise. Similarly, some inventions cause the price of products to fall dramatically (consider the invention of the electronic chip, for example), and in this case we might wish to evaluate how much consumers gain. In order to make such calculations, economists have developed the concept of *consumer surplus*, which permits welfare gains or losses to be estimated from knowledge of the market demand curve for a

[10] H. S. Houthakker, "Revealed Preference and the Utility Function," *Economica* 17 (May 1950): 159–174.

product. In this section we will show how these calculations are made; we will then use the consumer surplus notion in several places later in the text.

Consumer Welfare and Expenditure Functions

In Chapter 4 we developed the concept of the expenditure function as a way of recording the minimum expenditure necessary to achieve a desired level of utility given the prices of various goods. We denoted this function as

$$\text{expenditure} = E(P_X, P_Y, U_0), \tag{5.44}$$

where U_0 is the "target" level of utility that is sought. One way to evaluate the welfare cost of a price increase (say, from P_X^0 to P_X^1) would be to compare the expenditures required to achieve U_0 under these two situations:

$$\text{expenditures at } P_X^0 = E_0 = E(P_X^0, P_Y, U_0) \tag{5.45}$$

$$\text{expenditures at } P_X^1 = E_1 = E(P_X^1, P_Y, U_0), \tag{5.46}$$

so the loss in welfare would be measured as the increase in needed expenditures. Thus,

$$\text{welfare change} = E_0 - E_1. \tag{5.47}$$

Since $E_1 > E_0$, this change would be negative, indicating that the price rise makes this person worse off. On the other hand, if P_X fell, E_0 would exceed E_1 and the individual would experience a welfare gain. Knowledge of the expenditure function is therefore sufficient to make the kind of calculations we need.

A Graphical Approach

We can make further headway in this problem by using the envelope theorem result (see footnote 6 of this chapter) that the derivative of the expenditure function with respect to P_X yields the compensated demand function, h_X:

$$\frac{dE\,(P_X, P_Y, U_0)}{dP_X} = h_X(P_X, P_Y, U_0). \tag{5.48}$$

In words, the change in necessary expenditures brought about by a change in P_X is given by the quantity of X demanded. For evaluating this change in expenditures over a "large" price change (from P_X^0 to P_X^1), we must integrate Equation 5.48:

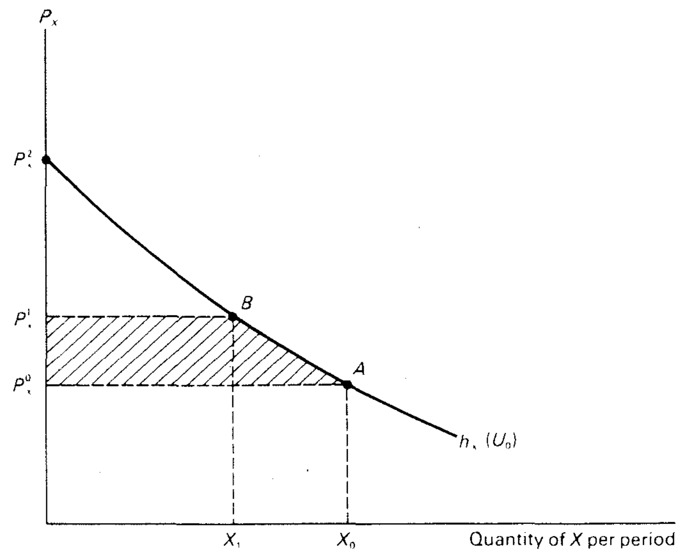
$$\text{change in expenditures} = \int_{P_X^0}^{P_X^1} dE = \int_{P_X^0}^{P_X^1} h_X(P_X, P_Y, U_0)\, dP_X \tag{5.49}$$

The integral in Equation 5.49 has a graphical interpretation—it is the area to the left of the compensated demand curve (h_X) between P_X^0 and P_X^1. This is our measure of welfare loss. It is illustrated as the shaded area between P_X^0

FIGURE 5.11

THE WELFARE LOSS OF A PRICE CHANGE

The shaded area to the left of the compensated demand curve, h_X, shows the amount that would have to be given to this individual to keep him or her as well-off at a price of P_X^1 as at a price of P_X^0. A consumer who buys X_0 at a price of P_X^0 receives a consumer surplus of $P_X^2AP_X^0$ since this is the increase in expenditures that would have to be provided to make this person willing to do without X completely.



and P_X^1 in Figure 5.11. For a fall in price below P_X^0, the welfare gain would be shown by a similar area below P_X^0.

Consumer Surplus

To understand the origin of the term *consumer surplus* to describe the welfare changes we have been examining, consider the following question: How much would the person whose demand curve is illustrated in Figure 5.11 need to be paid to voluntarily give up the right to consume X_0 at a price of P_X^0? A price of P_X^2 would be sufficiently high to prompt this person to reduce purchases of X to zero. Hence, by our previous discussion, it would require extra expenditures given by area $P_X^2AP_X^0$ to compensate this individual for doing without good X. Similarly, a person faced by the price P_X^0 chooses to consume X_0 and spends a total of $P_X^0 \cdot X_0$ on good X. In making these expenditures, he or she receives extra (or "surplus") welfare represented by the area $P_X^2AP_X^0$ relative to a situation in which X is not available at all. In our study of monopoly and other market imperfections, we will see how these often result in a loss of this consumer surplus or, in some cases, a transfer of consumer surplus from consumers to other market participants.

Welfare Changes and the Marshallian Demand Curve

So far, our graphic analysis of consumer surplus has made use of the compensated demand curve h_X. Because the location of this curve depends on the target level of utility assumed, there is some ambiguity about which curve to use. For example, in connection with Figure 5.11 we described the extra expenditures required to attain U_0 when good X costs P_X^1 rather than P_X^0. But in most actual applications this price rise will result in both substitution and income effects and a loss in utility to this individual (from, say, U_0 to U_1). That is, the actual market reaction to the rise in P_X would be to move from the point X_0, P_X^0 on the Marshallian demand curve (d_X) in Figure 5.12 to the point X_1, P_X^1 on that curve. At this new point, the individual will receive utility U_1, and for this level of utility the compensated demand curve is represented by $h_X(U_1)$ rather than the original curve, $h_X(U_0)$. The ambiguity then is whether the welfare loss is best described by the area $P_X^1BAP_X^0$ (as in Figure 5.11) or by the area $P_X^1CDP_X^0$ associated with the new curve, $h_X(U_1)$. Since the new area represents the reduction in expenditures that can be made in order to retain utility U_1 when the price of X falls from P_X^1 to P_X^0, it is unclear whether our original measure or this alternative measure more appropriately captures the change in welfare we seek to describe. It all depends on whether we assume that U_0 or U_1 is the appropriate utility target.

Fortunately, we have a compromise measure available. The size of the area to the left of the Marshallian demand curve between P_X^0 and P_X^1 (given by $P_X^1CAP_X^0$) clearly falls between the size of the welfare losses defined by $h_X(U_0)$ and $h_X(U_1)$. Since information in the Marshallian curve is also more likely to be available from real-world data, this seems a very good compromise indeed.[11] Of course, if the price changes we were examining were quite small, there would be little distinction among these three measures, and it is common in many economic discussions of welfare gains or losses to be rather imprecise about exactly what type of demand curve is being used for the analysis.

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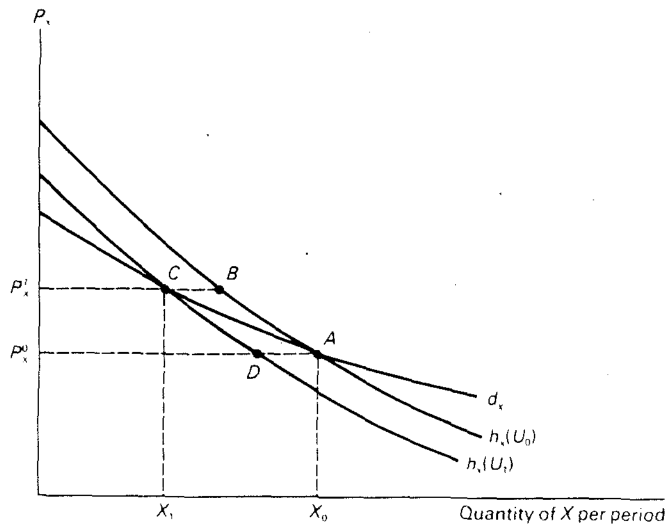
[11] For a further discussion, see R. D. Willig, "Consumer's Surplus without Apology," *American Economic Review* (September 1976): 589–597.

FIGURE 5.12

WELFARE EFFECTS OF PRICE CHANGES AND THE MARSHALLIAN DEMAND CURVE

d_X is the usual Marshallian (nominal income constant) demand curve for good X. $h_X(U_0)$ and $h_X(U_1)$ denote the compensated demand curves associated with the utility levels experienced when P_X^0 and P_X^1, respectively, prevail. The area to the left of d_X between P_X^0 and P_X^1 is bounded by the similar areas to the left of $h_X(U_0)$ and $h_X(U_1)$. Hence, for small changes in price, the area to the left of d_X is a good measure of welfare loss.



EXAMPLE 5.5

LOSS OF CONSUMER SURPLUS FROM A RISE IN SOFT DRINK PRICE

These ideas can be illustrated with our well-worn soft drink example. From Example 5.2 we know that the compensated demand function for soft drinks is given by

$$X = h_X(P_X, P_Y, V) = \frac{VP_Y^{.5}}{P_X^{.5}} \tag{5.50}$$

so, by Equation 5.49, the welfare loss from a price increase from $P_X = .25$ to $P_X = 1$ is given by

$$\textbf{change in welfare} = \int_{.25}^{1} \frac{VP_Y^{.5}\,dP_X}{P_X^{.5}}$$

$$= 2VP_Y^{.5}P_X^{.5}\Big|_{P_X=.25}^{P_X=1}. \qquad \textbf{(5.51)}$$

If we assume $V = 2$ is the initial utility level, this loss (since $P_Y = 1$) is given by

$$\textbf{loss} = 4(1)^{.5} - 4(.25)^{.5} = 2, \qquad \textbf{(5.52)}$$

which is exactly what we found in Example 5.3—when P_X rises to 1, expenditures must rise from 2 to 4 to keep this person from being made worse off. If the utility level experienced after the price rise is believed to be the more appropriate utility target for measuring the welfare loss, then $V = 1$ (see Example 5.3) and the loss would be given by

$$\textbf{loss} = 2(1)^{.5} - 2(.25)^{.5} = 1. \qquad \textbf{(5.53)}$$

If the loss were evaluated using the uncompensated (Marshallian) demand function

$$X = d_X(P_X, P_Y, I) = \frac{I}{2P_X}, \qquad \textbf{(5.54)}$$

the computation would be

$$\text{loss} = \int_{.25}^{1} \frac{I}{2P_X}\,dP_X$$

$$= I\frac{\ln P_X}{2}\Big|_{.25}^{1} = 0 - (-1.39) = 1.39, \qquad \textbf{(5.55)}$$

which does indeed represent a compromise between the two figures computed using the compensated functions.

QUERY: In this problem total consumer surplus cannot be computed since the demand curves are asymptomatic to the price axis and the required integrals do not converge. How might you make an approximation to total consumer surplus in this case? Or, is our analysis only valid for relatively small changes in P_X?